Coulomb-Blockade Oscillations in Semiconductor Nanostructures (Part I & II)

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Part I

- Introduction to Coulomb-blockade oscillations
- Basic properties of semiconductor nanostructures

Introduction

- Coulomb-blockade Oscillations: A manifestation of single-electron tunneling through a system of two tunnel junctions in series.
- They occur when the voltage on a nearby gate electrode is varied.
- Tunneling is blocked at low temperatures where the charge imbalance jumps from + e/2 to – e/2 (except near the degeneracy points).

Introduction

- Semiconductor nanostructures are fabricated by lateral confinement of the 2DEG in Si-inversion layers, or in GaAs-AlGaAs heterostructures.

- First type of semiconductor nanostructure found to exhibit Coulomb-blockade oscillations: **A narrow disordered wire**, defined by a split-gate technique.

- Second type of semiconductor nanostructure: A small artificially confined region in a 2DEG (a quantum dot), connected by tunnel barriers.

- Electrons in a 2DEG move in a plane due to a strong electrostatic confinement at the interface between two semiconductor layers or a semiconductor and an insulator.

- The areal density can be continuously varied by changing the voltage on a gate electrode deposited on the top semiconductor layer or on the insulator.

- The gate voltage is defined with respect to an ohmic contact to the 2DEG.

- The density under a gate electrode of large area changes linearly with the electrostatic potential of the gate φ_{gate}, according to the plate capacitor formula:

$$\delta n_s = \frac{\varepsilon}{ed}\delta\varphi_{gate}$$

- A unique feature of a 2DEG is that it can be given any desired shape using lithographic techniques.

- The energy of non-interacting conduction electrons in an unbounded 2DEG is given by:

$$E_{(k)} = \frac{\hbar^2 k^2}{2m}$$

- The density of states per unit area is independent of the energy:

$$\rho_{2D} = g_s g_v \frac{m}{2\pi\hbar^2}$$

where g_s and g_v account for the spin and valley-degeneracy.

- In equilibrium, the states are occupied according to the Fermi-Dirac distribution function:

$$f(E - E_F) = \left[1 + \exp\left(\frac{E - E_F}{k_B T}\right)\right]^{-1}$$

- At low temperatures $k_B T \ll E_F$, the Fermi energy E_F of a 2DEG is directly proportional to its sheet density n_s, according to:

$$E_F = \frac{n_s}{\rho_{2D}}$$

- The Fermi wave number $k_F = \left(2mE_F / \hbar^2\right)^{1/2}$ is related to the density by:

$$k_F = (4\pi n_s / g_s g_v)^{1/2}$$

- When 2DEG is confined laterally to a narrow channel, its conduction band splits itself into a series of one-dimensional subbands. The total energy of *En(k)* of an electron in the *n*-th 1D subband in zero magnetic field is given by:

$$E_n(k) = E_n + \frac{\hbar^2 k^2}{2m}$$

- Two frequently used potentials to model analytically the lateral confinement are square well potential and the parabolic potential well. The confinement levels are given by:

$$E_n = (n\pi\hbar)^2 / 2mW^2, \text{ and } E_n = (n - \frac{1}{2})\hbar\omega_0$$

- Transport through a very short quantum wire (~ 100 nm, much shorter than the mean free path) is perfectly **ballistic**: quantum point contact.

- The conductance *G* of a quantum point contact is quantized in units of $2e^2/h$. This effect requires a unit transmission probability for all of the occupied 1D sub-bands in the point contact, each of which then contributes $2e^2/h$ to the conductance ($g_s g_v = 2$).

- Quantum wires are extremely sensitive to disorder, since the effective scattering cross-section, being of the order of Fermi wavelength, is comparable to the width of the wire.

- A quantum dot is formed in a 2DEG if the electrons are confined in all three directions and its energy spectrum is fully discrete.
- Transport through the discrete states in a quantum dot can be studied if tunnel barriers are defined at its perimeter.
- The quantum point contacts are operated close to pinch-off ($G < 2e^2/h$), where they behave as tunnel barriers of adjustable height and width.
- The shape of such barriers differs from that encountered in metallic tunnel junctions.

Part II

Theory of Coulomb-blockade oscillations

- Periodicity of the oscillations
- Amplitude and lineshape

Periodicity of the oscillations

- In a weakly coupled quantum dot, transport proceeds by tunneling through its discrete electronic states.

- In the absence of charging effects, a conductance peak due to resonant tunneling occurs when the Fermi energy E_F in the reservoirs lines up with one of the energy levels in the dot.

- The probability to find N electrons in the quantum dot in equilibrium with the reservoirs is given by:

$$P(N) = \text{constant x} \exp\left(-\frac{1}{k_B T}\left[F(N) - NE_F\right]\right)$$

Periodicity of the oscillations

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- F(N) is the free energy of the dot, T is the temperature, E_F is the reservoir Fermi energy.

- At T=0, P(N) is non-zero for only a single value of N (for the value which minimizes the thermodynamic potential $\Omega(N) = F(N) - NE_F$).

- A non-zero G (conductance) is possible only if P(N) and P(N+1) are both non-zero for some N. A small applied voltage is then sufficient to induce a current through the dot.

- To have P(N) and P(N+1) both non-zero at T=0 requires that both N and N+1 minimize the thermodynamic potential in way that $F(N+1) - F(N) = E_F$.

Periodicity of the oscillations

- At T=0, the free energy F(N) equals the ground state energy of the dot, for which we take the simplified form:

$$F(N) = U(N) + \sum_{p=1}^{N} E_p$$

Here U(N) is the charging energy, and E_p (p=1,2,…) are single-electron energy levels in ascending order. The term U(N) accounts for the charge imbalance between dot and reservoirs.

- The sum over energy levels accounts for the internal degrees of freedom of the quantum dot, evaluated in a mean-field approximation.

Periodicity of the oscillations

- Each level contains either one or zero electrons. The energy levels Ep depend on gate voltage and magnetic field, but are assumed to be independent of N. A peak in the low temperature conductance occurs whenever:

$$E_N + U(N) - U(N-1) = E_F$$

Here, U(N) is written as:

$$U(N) = \int_0^{-Ne} \varphi(Q)' dQ' \text{ where } \varphi(\mathrm{Q}) = \mathrm{Q}/\mathrm{C} + \varphi_{\mathrm{ext}}$$

Periodicity of the oscillations

- The capacitance C is assumed to be independent of N and the charging energy then takes the form:

$$U(N) = (Ne)^2 / 2C - Ne\varphi_{ext.}$$

- The periodicity is given by the equation:

$$\Delta E_F = \Delta E^* \equiv \Delta E + \frac{e^2}{C}$$

In the absence of charging effects, ΔEF is determined by the irregular spacing ΔE of the single electron levels in the quantum dot.

Periodicity of the oscillations

- To determine the periodicity in case of Coulomb-blockade oscillations, we need to know how E_F and the set of energy levels E_P depend on φ_{ext}.

- In a 2DEG, the external charges are supplied by ionized donors and by a gate electrode (with an electrostatic potential difference φ_{gate} between gate and 2DEG reservoir) and can be expressed as:

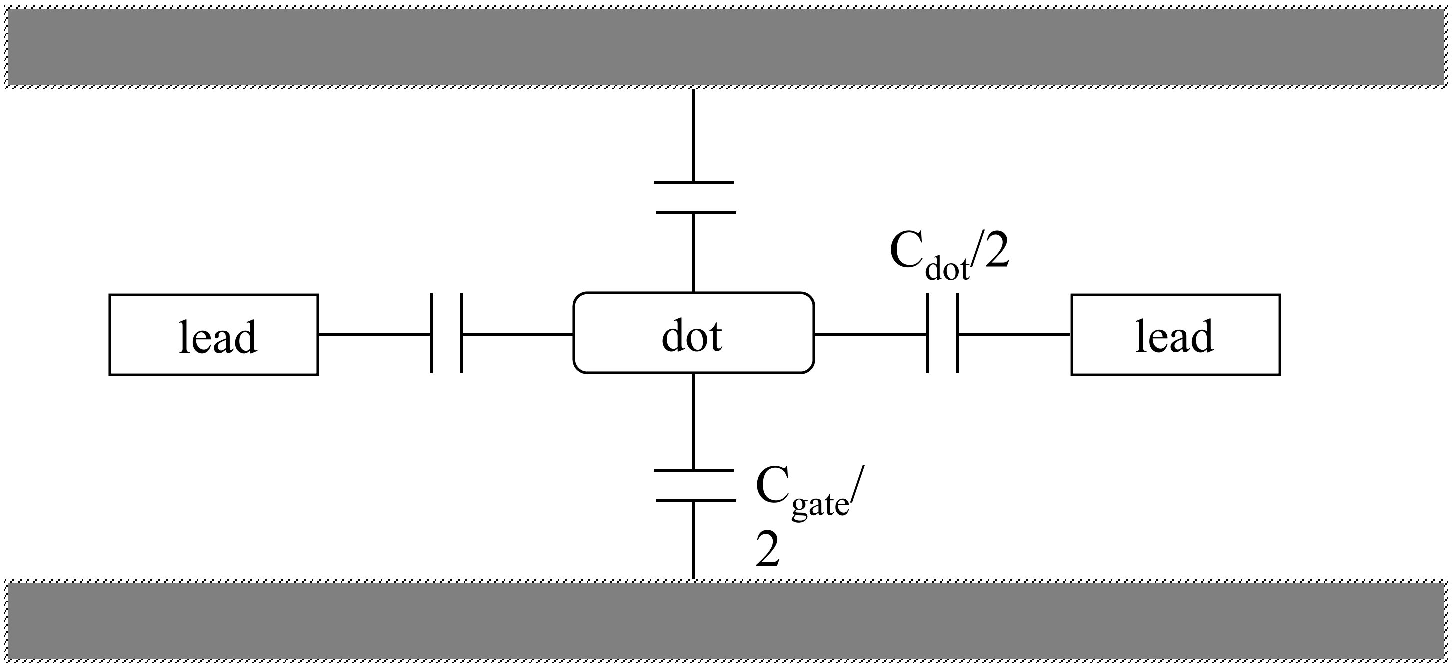
$$\varphi_{ext} = \varphi_{donors} + \alpha\varphi_{gate}$$

- The period of the oscillations can expressed as:

$$\Delta\varphi_{gate} = \frac{e}{\alpha C}$$

Where α is a rational function of the capacitance matrix elements of the system And depends on the geometry.

Periodicity of the oscillations



Equivalent circuit of quantum dot and split gate. The mutual capacitance of leads and gate is much larger than that of the dot and the split gate.

Periodicity of the oscillations

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- The gate voltage V_{gate} is the electrochemical potential difference between gate and leads. The oscillation period ΔV_{gate} is given by:

$$\Delta V_{gate} = \frac{\Delta E}{e} + \Delta \varphi_{gate} = \frac{\Delta E}{e} + \frac{e}{C_{gate}}$$

Amplitude and lineshape

- The equilibrium distribution function of electrons among the energy levels is given by the Gibbs distribution in the grand canonical ensemble:

$$P_{eq}(\{n_i\}) = \frac{1}{Z}\exp\left[-\frac{1}{k_B T}\left(\sum_{i=1}^{\infty} E_i n_i + U(N) - NE_F\right)\right]$$

- Where {ni}={n1, n2, …} denotes a specific set of occupation numbers of the energy levels in quantum dot. The number of electrons in the dot is N=∑ni and Z is the partition function:

$$Z = \sum_{\{n_i\}}\exp\left[-\frac{1}{k_B T}\left(\sum_{i=1}^{\infty} E_i n_i + U(N) - NE_F\right)\right]$$

Amplitude and lineshape

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- The joint probability P_{eq} (N, n_p=1) that the quantum dot contains N electrons and that level p is occupied is:

$$P_{eq}(N, n_p = 1) = \sum_{\{n_i\}} P_{eq}(\{ni\}) \delta_{N,\sum n_i} \delta_{n_p,1}$$

- In terms of this probability, the conductance is given by:

$$G = \frac{e^2}{k_B T} \sum_{p=1}^{\infty} \sum_{N=1}^{\infty} \frac{\Gamma_p^l \Gamma_p^r}{\Gamma_p^l + \Gamma_p^r} P_{eq}(N, n_p = 1) \, \mathrm{X} \left[1 - f(E_p + U(N) - U(N-1) - E_F)\right]$$

Amplitude and lineshape

- The conductance of the quantum dot in the high temperature limit is simply that of the two tunnel barriers in series:

$$G = \frac{G^l G^r}{G^l + G^r}, \text{ if } \Delta E, e^2 / C << k_B T << E_F$$

- The conductances G^l and G^r of the left and the right tunnel barriers are given by the thermally averaged Landauer formula:

$$G^{l,r} = -\frac{e^2}{r}\int_0^{\infty} dE T^{l,r}(E)\frac{df}{dE}$$

Amplitude and lineshape

- The transmission probability of a barrier T(E) equals the tunnel rate Γ(E) divided by the attempt frequency ν(E)=1/hϱ(E):

$$T^{l,r}(E) = h\Gamma^{l,r}(E)\rho(E)$$

- If the height of the tunnel barriers is large, the energy dependence of the tunnel rates and of the density of states in the dot can be ignored. The conductance of each barrier the becomes:

$$G^{l,r} = (e^2/h)T^{l,r} = e^2\Gamma^{l,r}\rho$$

Amplitude and lineshape

- The conductance of the quantum dot becomes:

$$G = e^2 \rho \frac{\Gamma^l \Gamma^r}{\Gamma^l + \Gamma^r} = \frac{e^2}{r} \frac{T^l T^r}{T^l + T^r} \equiv G_\infty$$

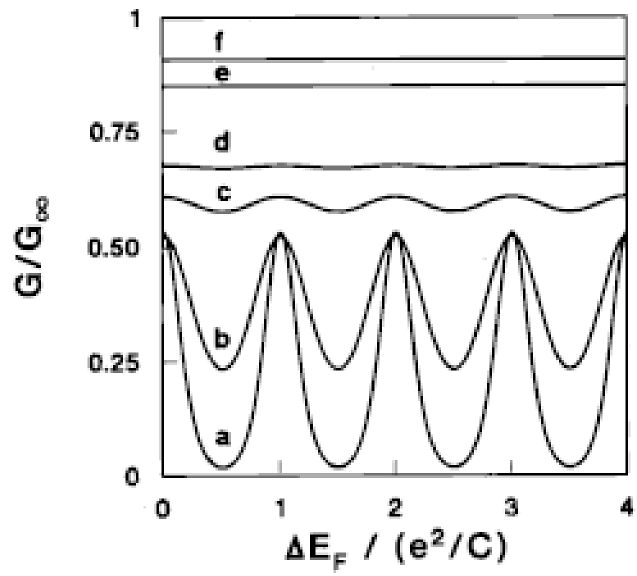
If ΔE, $e^2/C << k_B T << E_F$

- For energy-independent tunnel rates and density of states, one obtains a line shape of individual conductance peaks given by:

$$G / G_{\max} = \frac{\Delta_{\min} / k_B T}{\sinh(\Delta_{\min} / k_B T)} \approx \cosh^{-2}\left(\frac{\Delta_{\min}}{2.5 k_B T}\right)$$

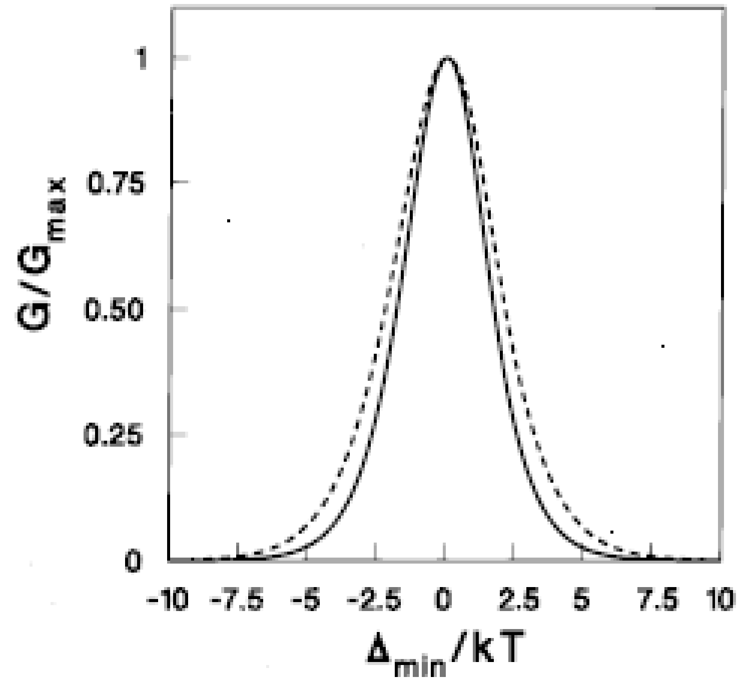
Amplitude and lineshape

- The width of the peaks increases with T in the classical regime, whereas the peak height is temperature independent.



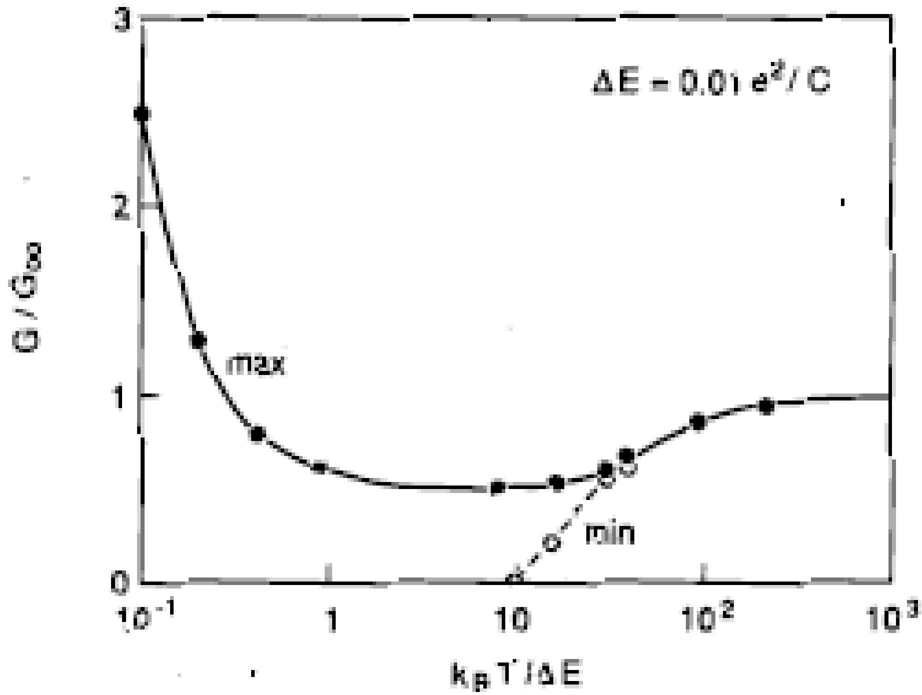
Temperature dependence of the Coulomb-blockade oscillations as a function of Fermi energy in the classical regime.

Amplitude and lineshape



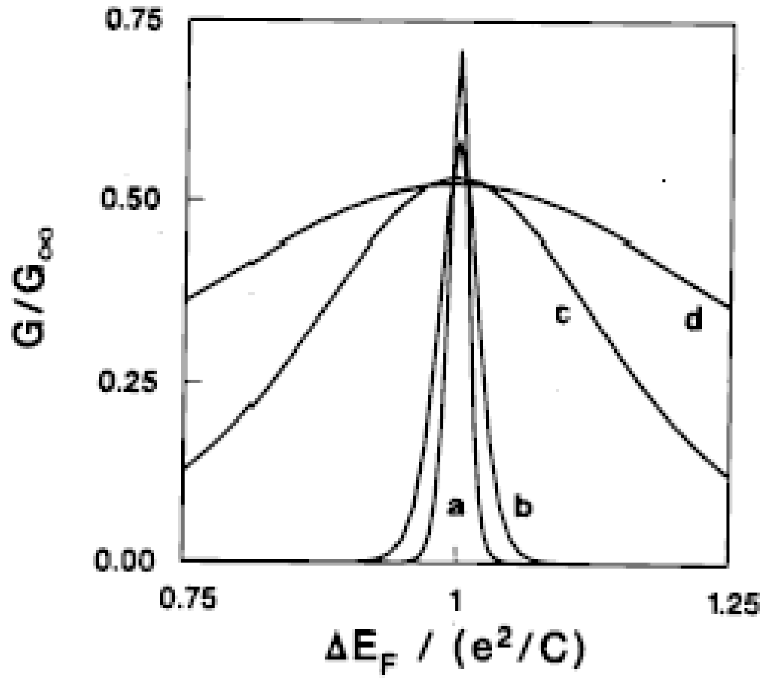
Comparison of the lineshape of a thermally broadened conductance peak in the resonant tunneling regime.

Amplitude and lineshape



Temperature dependence of the maxima (max) and the minima (min) of the Coulomb-blockade oscillations.

Amplitude and lineshape



Lineshape for various temperatures, showing the crossover from the resonant tunneling regime (a and b) to the classical regime (c and d).

Thank You