

Klein Tunneling

PHYS 503 Physics Colloquium Fall 2008

9/11

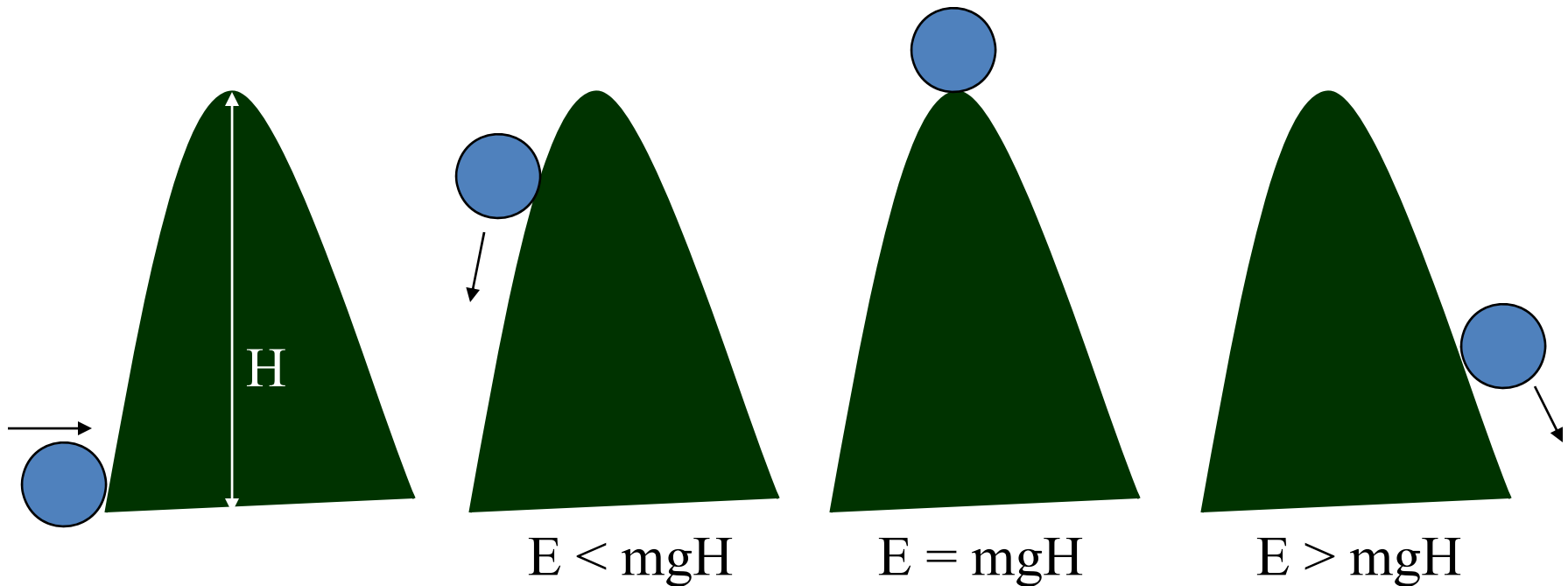
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Outline

- Classical picture
- Tunneling
- Klein Tunneling
- Bipolar junctions with graphene
- Applications

Classical Picture



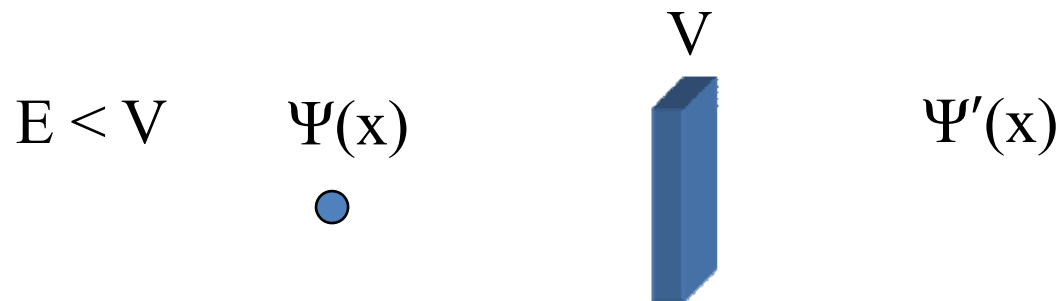
Kinetic Energy = E
Mass of the ball = m

- Transmission of a particle through a potential barrier higher than its kinetic energy ($V > E$).
- It violates the principles of classical mechanics.
- It is a **quantum** effect.

- On the quantum scale, objects exhibit *wave-like* characteristics.
- Quanta moving against a potential hill can be described by their wave function.
- The wave function represents the *probability amplitude* of finding the object in a particular location.

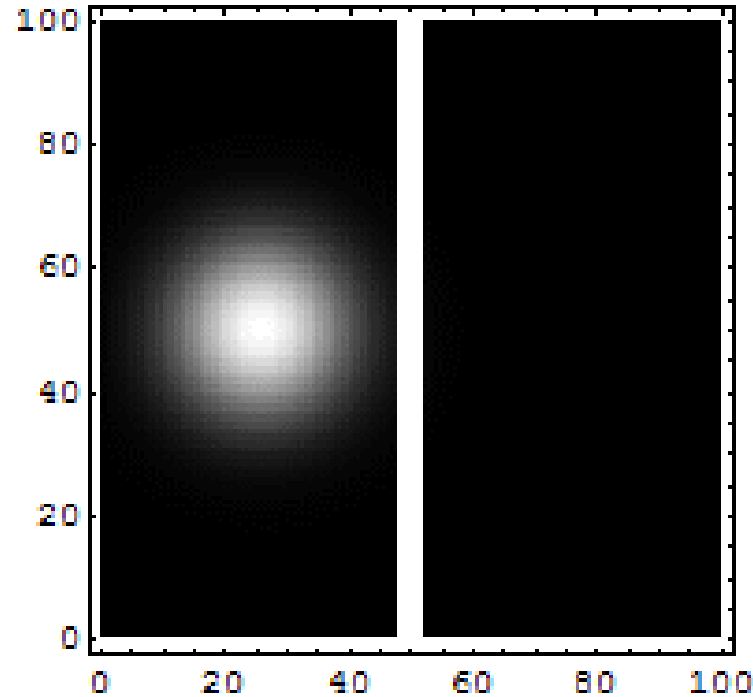
Quantum tunneling effect

- If this wave-function describes the object as being on the other side of the potential hill, then there is a probability that the object has moved through the potential hill.
- This transmission of the object through the potential hill is termed as *tunneling*.



Tunneling = Transmission through the potential barrier

Tunneling



Reflection —→ Interference fringes
Transmission —→ Tunneling

- In quantum mechanics, an electron can tunnel from the conduction into the valence band.
- Such tunneling from an electron-like to hole-like state is called as interband tunneling or *Klein* tunneling.
- Here, electron avoids backscattering

Tunneling in Graphene

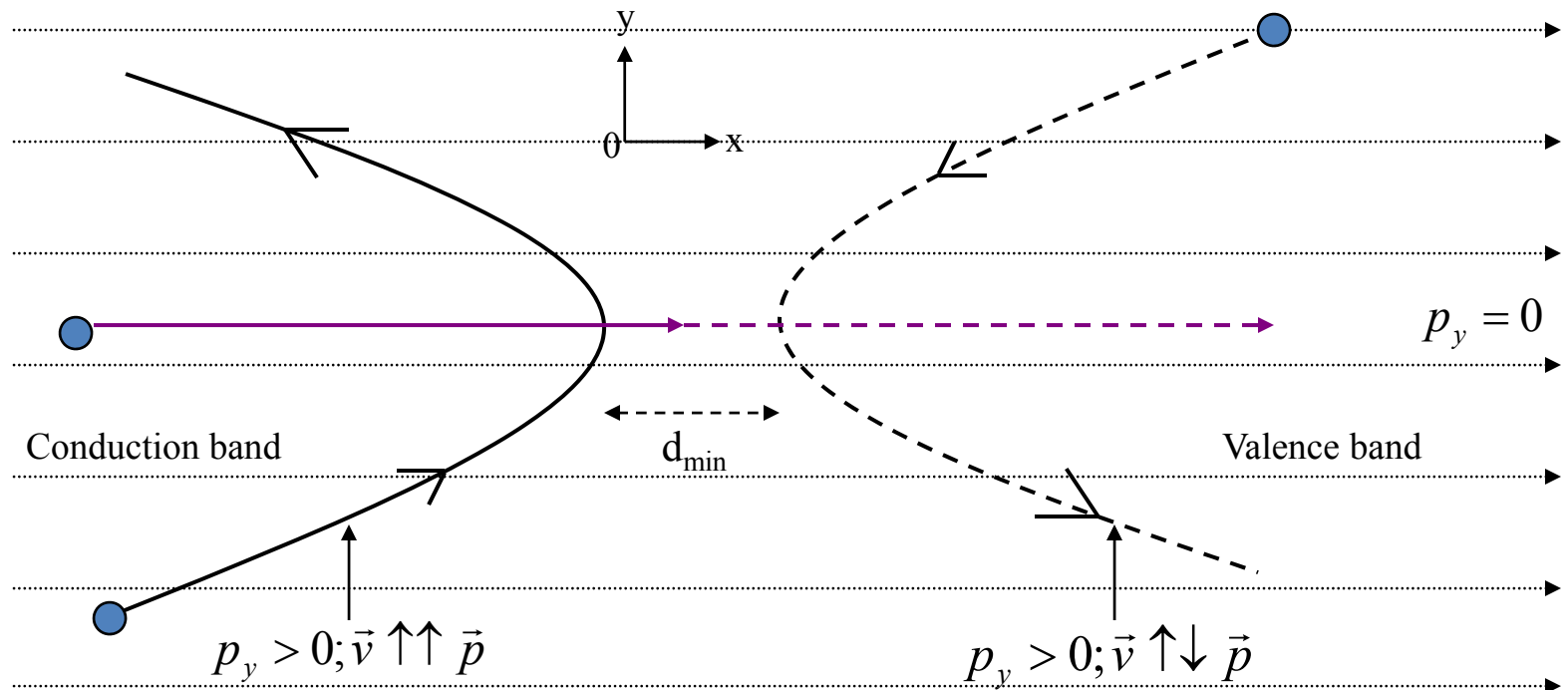
- In graphene, the massless carriers behave differently than ordinary massive carriers in the presence of an electric field.
- Here, electrons avoid backscattering because the carrier velocity is *independent* of the energy.
- The absence of backscattering is responsible for the high conductivity in carbon nanotubes (Ando et al, 1998).

Absence of backscattering

Let's consider a linear electrostatic potential

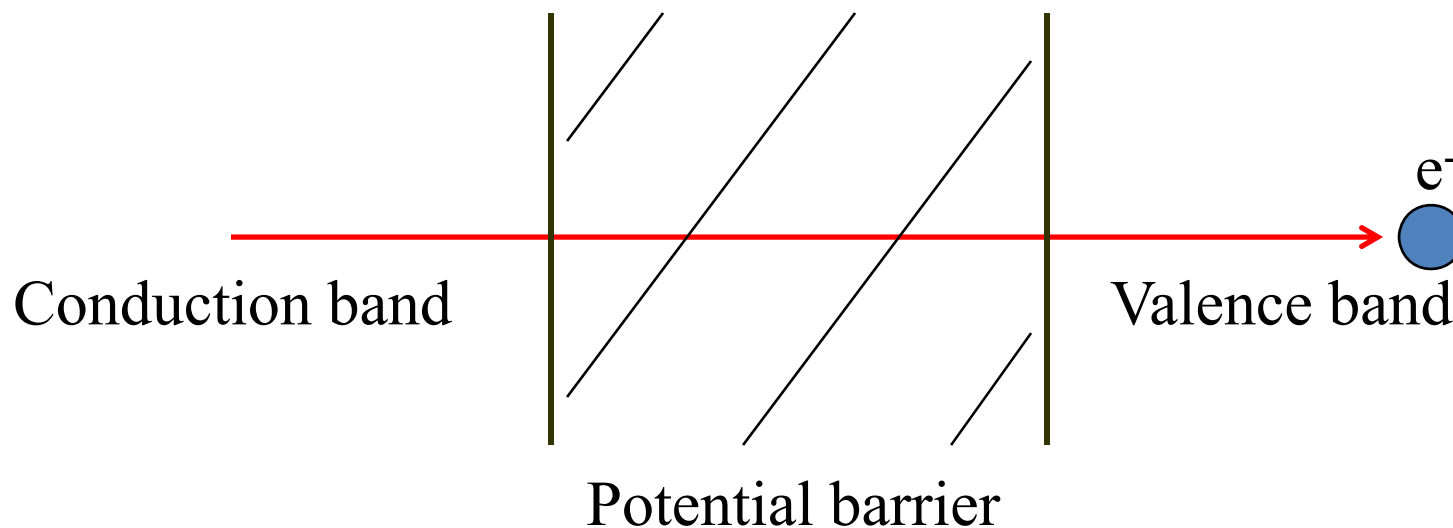
$$U(x) = Fx$$

Electron trajectories will be like:

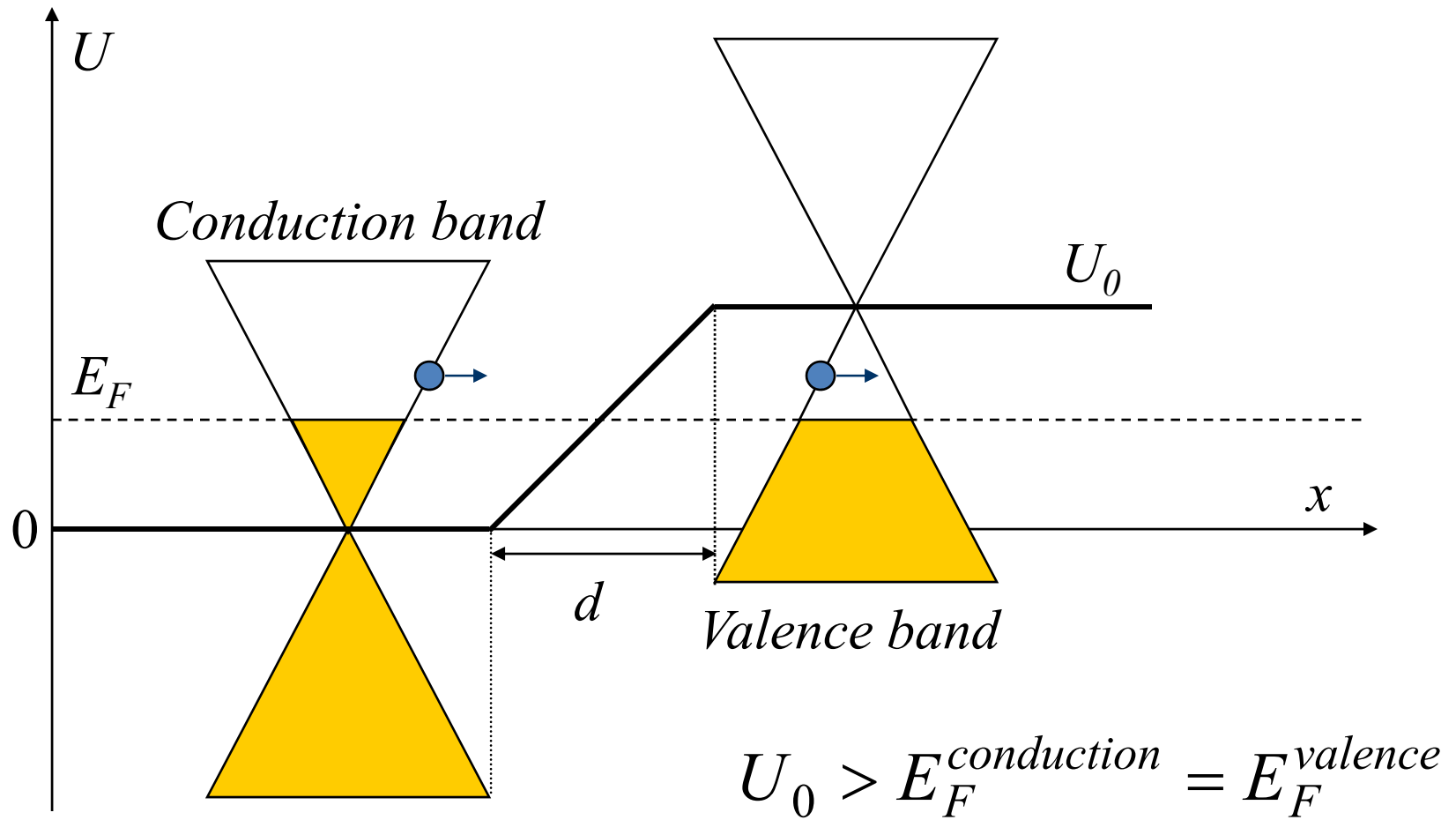


Absence of backscattering

- For $p_y = 0$, *no* backscattering.
- The electron is able to propagate through an infinitely high potential barrier because it makes a *transition* from the conduction band to the valence band.



Band structure



Absence of backscattering

- In this transition from conduction band to valence band, its dynamics changes from electron-like to hole-like.
- The equation of motion is thus,

$$\frac{d\vec{r}}{dt} \equiv \frac{\partial E}{\partial \vec{p}} = \frac{v^2 \vec{p}}{E - U}$$

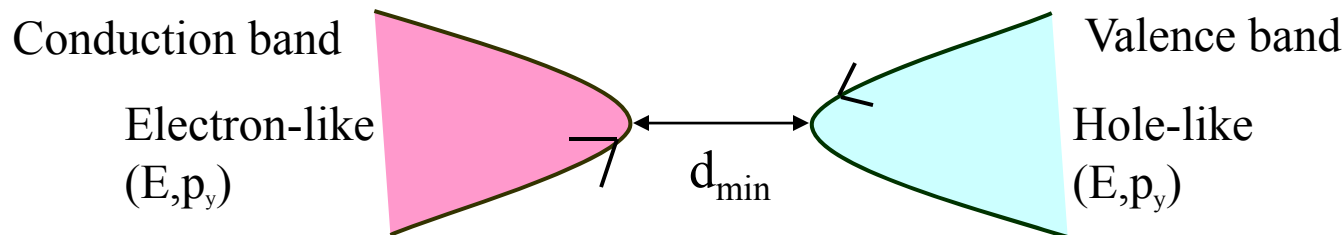
at energy E with $v^2 |\vec{p}|^2 = (E - U)^2$

- It shows that $\vec{v} \uparrow \uparrow \vec{p}$ in the conduction band ($U < E$) and $\vec{v} \uparrow \downarrow \vec{p}$ in the valence band ($U > E$).

Klein tunneling

- States with $\vec{v} \uparrow \uparrow \vec{p}$ are called electron-like.
- States with $\vec{v} \uparrow \downarrow \vec{p}$ are called hole-like.
- Pairs of electron-like and hole-like trajectories at the same E and p_y have turning points at:

$$d_{\min} = \frac{2v |p_y|}{F}$$



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- The tunneling probability: exponential dependence on d_{\min} .

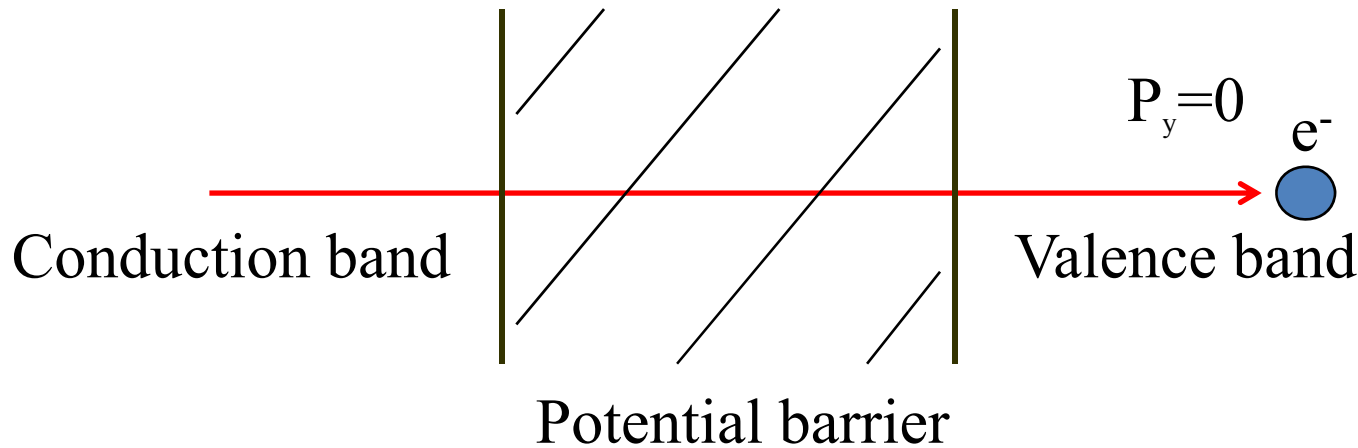
$$T(p_y) = \exp\left(\frac{-\pi |p_y| d_{\min}}{2\hbar}\right) = \exp\left(\frac{-\pi v p_y^2}{\hbar F}\right)$$

Condition: p_x^{in} at $x \rightarrow -\infty$ and p_x^{out} at $x \rightarrow \infty$ is sufficiently large :

$$|p_x^{in}|, |p_x^{out}| \gg |p_y|, \sqrt{\frac{\hbar F}{v}}$$

Transmission resonance

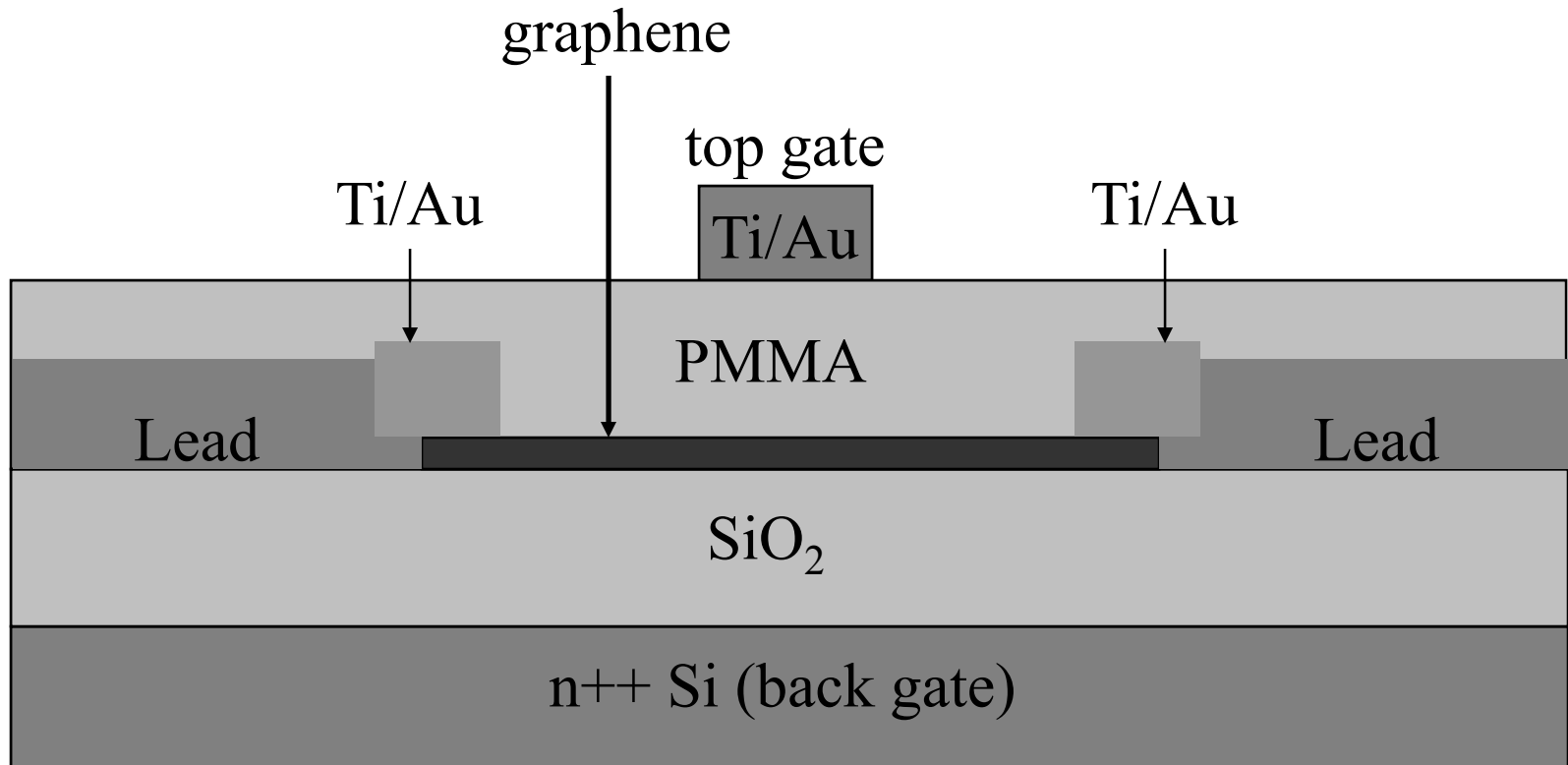
- It occurs when a p-n junction and an n-p junction form a p-n-p or n-p-n junction.
- At $p_y=0$, $T(p_y)=1$ (unit transmission): No transmission resonance at normal incidence.



No transmission resonance

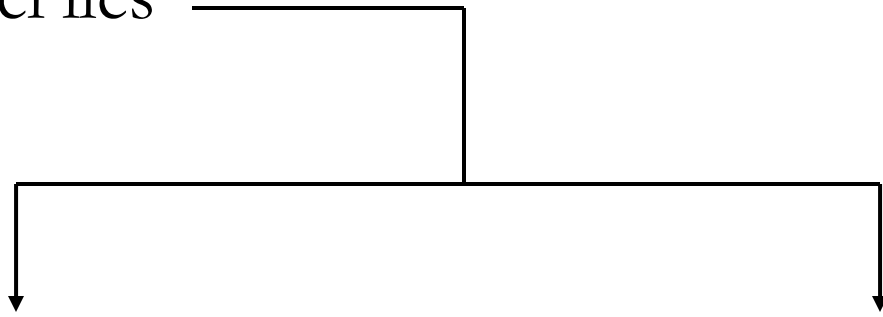
Bipolar junctions

- Electrical conductance through the interface between p-doped and n-doped graphene: **Klein tunneling**.



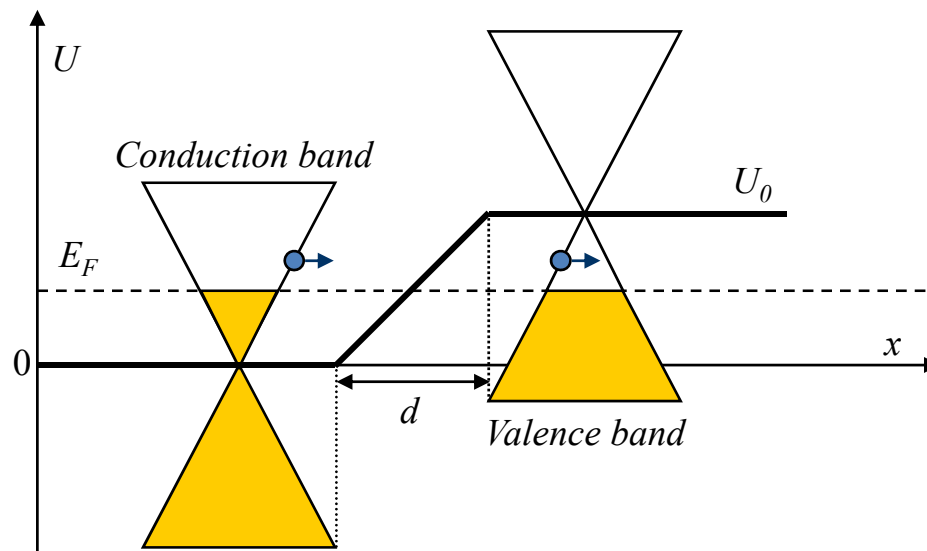
Bipolar junctions

- Top gate: Electrostatic potential barrier
- Fermi level lies



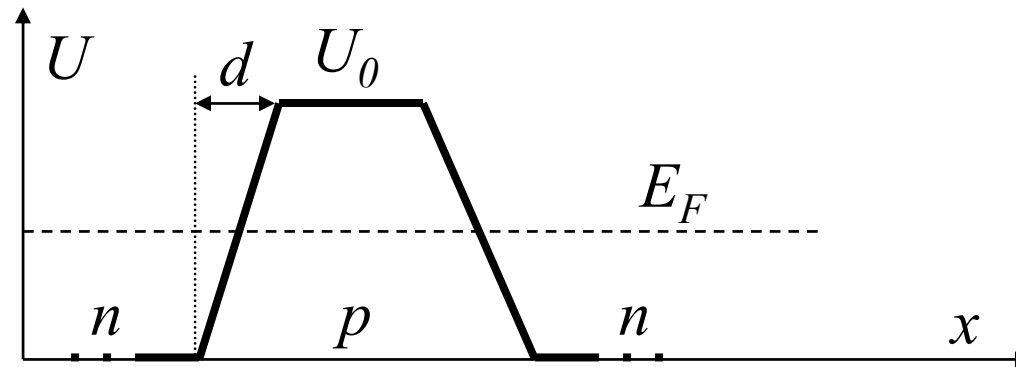
In the *Valence band* inside the barrier
 (*p*-doped region)

In the *Conduction band* outside the barrier
 (*n*-doped region)



Bipolar junctions

- Carrier density $n_{carrier}$ is the same in the n and p regions when the Fermi energy is half the barrier height U_0 .



Fermi momenta in both the n and p regions are given by:

$$p_F \equiv \hbar k_F = \frac{U_0}{2v} = \frac{Fd}{2v}$$

$$d \approx 80 \text{ nm}^\dagger$$

[†] Measured by Huard et al (2007) for their device.

- The Fermi wave vector ($k_F = \sqrt{\pi n_{carrier}}$) for typical carrier densities of $n_{carrier} \geq 10^{12} \text{ cm}^{-2}$ is $> 10^{-1} \text{ nm}^{-1}$.
- Under these conditions $k_F d > 1$, p - n and n - p junctions are smooth on the Fermi wavelength.
- The tunneling probability expression can be used.

$$T(p_y) = \exp\left(\frac{-\pi |p_y| d_{\min}}{2\hbar}\right) = \exp\left(\frac{-\pi v p_y^2}{\hbar F}\right)$$

Bipolar junctions

- The conductance G_{p-n} of a $p-n$ interface can be solved by integration of tunneling probability over the transverse momenta
- The result of integration †:

$$G_{p-n} = \frac{4e^2}{h} \frac{W}{2\pi\hbar} \int_{-\infty}^{\infty} dp_y T(p_y) = \frac{4e^2}{h} \frac{W}{2\pi} \sqrt{\frac{F}{\hbar v}}$$

where W is the transverse dimension of the interface.

† Cheianov and Fal'ko, 2006

Applications of tunneling

- Atomic clock
- Scanning Tunneling Microscope
- Tunneling diode
- Tunneling transistor

Questions ?

Who got the Nobel prize (1973) in Physics for his pioneering work on electron tunneling in solids?



Dr. Leo Esaki (b. 1925, Osaka, Japan)

Thanks !!